V. CONCLUSIONS

Using the impedance matrix formulation, the circulator parameters and performance characteristics are determined. The relative bandwidth at port 3 is less than that at port 4. These bandwidths increase slightly with increasing the stripline coupling angle.

A 4-port circulator is realized by the interaction of many space harmonics. The zero mode has a relatively large value and consequently it greatly affects the circulation action. The electric field at the center of the ferrite disk is strong. Therefore, the insertion of a central metal post will greatly disturb the circulation condition.

The study of the power-density distribution may be useful for circulators operating at high-power levels. It is noticed that the power is mainly concentrated near and between ports 1 and 2.

An inner dielectric would not only improve the power capability of the circulator, but would provide more flexibility in the design as well.

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Insertion Loss of 3-Port Circulator with One Port Terminated in Variable Short Circuit

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Abstract—**The insertion loss between ports 1 and 3 of a 3-port circulator with port 2 terminated in a short circuit varies about twice the single path loss. The purpose is to give approximate simple upper and lower bounds for this loss in terms of the single path insertion loss of the junction. One application of this arrangement is encountered in the connection of a filter and equalizer by a single circulator. The final result indicates that, in the absence of circuit losses, the double path loss varies between one and three times the single path loss.**

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The insertion loss of a 3-port circulator between ports 1 and 3 with port 2 terminated in a variable short circuit is of some interest in a number of situations. One such application is where a circulator is used to connect a filter and an equalizer. Another application is in phasors using circulators with one port terminated in a p-i-n diode switch. The insertion loss of such an arrangement has been studied in [1] and [2], in terms of the elements of the Q-matrix. The purpose of this short paper is to give approximate upper and lower bounds on the loss in terms of the single path loss L_0 of the circulator.

The exact relation is given in [1] by

$$L = \frac{P_a}{P_i} = X(1 + S_{21}^2) - 2YS_{21}\cos(\eta - \Psi_1 + \Psi_2) \qquad (1)$$

where L is the total insertion loss between ports 1 and 3, P_i and P_a are the total incident and dissipated powers, S_{21} is the usual transmission coefficient between ports 1 and 2, $\Psi_{1,2}$ are the phase angles of the input waves at ports 1 and 2, X and $Ye^{j\eta}$ are the entries of the Q-matrix, and X represents the single path insertion loss of the circulator between ports 1 and 2. Fig. 1 depicts the schematic studied in this text. Some simplification of the preceding relation is possible by relating X, Y, and S_{21} through the eigenvalues of the Q-matrix [3].

The result is

$$X \approx \frac{2q_1}{3} \qquad (2)$$

$$Y \approx \frac{q_1}{3} \qquad (3)$$

$$S_{21} \approx 1 - \frac{q_1}{3} \qquad (4)$$

$$\eta = 0 \qquad (5)$$

provided

$$S_{11} \approx S_{12} \approx 0 \qquad (6)$$

where q_1 is the dissipation eigenvalue of the demagnetized counterrotating eigennetworks.

It is observed that $X = 2Y$ is consistent with the example given by Hagelin in [1].

The preceding approximations omit the dissipation of the in-phase

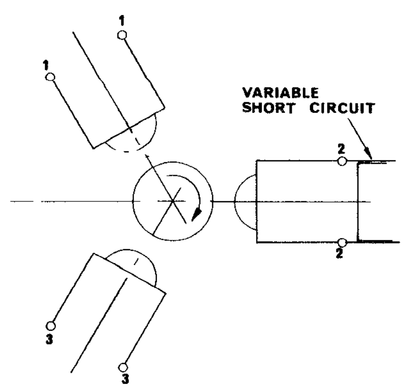

Fig. 1.

TABLE I

F_0(GHz)	L_0(dB)	L_{min}(dB)	L_{max}(dB)	$L_{0'} = \frac{L_{max} + L_{min}}{4}$	$L_F = \frac{L_{max} - L_{min}}{2}$	$L_C = L_{0'} - L_F$	Ferrite Material (Trans-Tech)
4.4	0.16	0.24	0.37	0.15	0.065	0.085	G1004
4.6	0.14	0.25	0.36	0.152	0.055	0.097	G1004
4.8	0.15	0.26	0.34	0.147	0.040	0.107	G1004
9.0	0.18	0.32	0.44	0.19	0.06	0.13	TT1-390
10.5	0.186	0.29	0.47	0.192	0.09	0.101	TT1-390
9.0	0.17	0.31	0.37	0.172	0.03	0.14	G1001

eigennetwork, and furthermore assume that the dissipation of the two counterrotating eigennetworks is equal.

Substituting the preceding relations into (1) gives

$$L = L_0\left[1 + \left(1 - \frac{L_0}{2}\right)^2\right] - L_0\left(1 - \frac{L_0}{2}\right)\cos\left(\Psi_1 + \Psi_2\right) \tag{7}$$

where we have put

$$X = L_0.$$

The theoretical minimum and maximum values of the loss are therefore

$$L_{\min} = L_0\left[1 + \left(1 - \frac{L_0}{2}\right)^2\right] - L_0\left(1 - \frac{L_0}{2}\right) \tag{8}$$

$$L_{\max} = L_0\left[1 + \left(1 - \frac{L_0}{2}\right)^2\right] + L_0\left(1 - \frac{L_0}{2}\right). \tag{9}$$

For instance, when $L_0 = 0.50$ dB the result is

$$L_{\min} = 0.475 \text{ dB}$$

$$L_{\max} = 1.445 \text{ dB}.$$

Equations (8) and (9) suggest that for L_0 small L varies between L_0 and $3L_0$.

The following results have been obtained for a waveguide circulator consisting of a simple lossy ferrite post at the junction of three rectangular waveguides:

$$L_0 = 0.36 \text{ dB}$$

$$L_{\min} = 0.38 \text{ dB}$$

and

$$L_{\max} = 1.14 \text{ dB}.$$

In the presence of circuit losses the following empirical relations apply:

$$L_{\min} \approx 2(L_C + L_F) - L_F \tag{10}$$

$$L_{\max} \approx 2(L_C + L_F) + L_F \tag{11}$$

where L_F is the single path ferrite loss and L_C is the single path circuit loss. The first term in the preceding two equations represents twice the single path loss which is consistent with the observation in [5].

This experiment may therefore be employed to separate ferrite and circuit losses in 3-port junction circulators—some experimental results which apply to quarter-wave coupled waveguide circulators at C and X bands are given in Table I. The circuit losses obtained here are consistent with those given in [4].

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Application of a Property of the Airy Function to Fiber Optics

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***Abstract*—The integral of the square of the Airy function from one of its zeros to infinity is equal to the square of the first derivative of the Airy function at the zero considered. Two important applications of this result to fiber optics are discussed.**

The Airy function is involved in many problems of fiber optics. For example, waves guided along the curved boundary of a homogeneous dielectric [1] (whispering gallery modes [2]) or along the straight boundary of a medium with constant transverse gradient of refractive index [3], are described by Airy functions. We shall show that the normalized field at the dielectric boundary is given by a very simple expression because of a property of the Airy function that does not seem to be known. Knowledge of the normalized field is essential to evaluate the coupling strength and the bending loss of a mode.

The Airy function Ai(x) is a solution of the differential equation [4]

$$d^2\,\text{Ai}\,(x)/dx^2 = x\,\text{Ai}\,(x). \tag{1}$$

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